## Cambridge O Level


#### Abstract

ADDITIONAL MATHEMATICS 4037/21 Paper 2 May/June 2020 MARK SCHEME Maximum Mark: 80


## Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.
This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.
Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE ${ }^{\text {TM }}$ and Cambridge International A \& AS Level components, and some Cambridge O Level components.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3:
Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

## Maths-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

## MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

## Types of mark

M Method marks, awarded for a valid method applied to the problem.
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

## Abbreviations

| awrt | answers which round to |
| :--- | :--- |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| nfww | not from wrong working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 1 | Valid method to find $m$ $m=\frac{34-9}{3-0.5}[=10]$ oe | M1 |  |
|  | Valid method to find $c$, e.g. $34=$ their $10 \times 3+c$ | M1 |  |
|  | $\sqrt[4]{y}=($ their 10$) \frac{1}{x}+$ their 4 | M1 |  |
|  | $y=\left(\frac{10}{x}+4\right)^{4}$ oe, cao | A1 |  |
| 2(a) | $9\left(x-\frac{2}{3}\right)^{2}+1$ oe | B3 | B1 for each of $p, q, r$ correct in correctly formatted expression; allow correct equivalent values <br> If B0 then $\mathbf{S C} 2$ for $9\left(x-\frac{2}{3}\right)+1$ or SC1 for correct values but other incorrect format |
| 2(b) | their $\left(\frac{2}{3}, 1\right)$ oe | B1 | FT their (a) |
| 3(a) | Finds p (-1) | M1 |  |
|  | 24 | A1 |  |
| 3(b)(i) | $\begin{aligned} & \mathrm{p}(-2)= \\ & 15(-8)+22(4)-15(-2)+2=0 \end{aligned}$ | B1 |  |
| 3(b)(ii) | Attempt to find the quadratic factor | M1 |  |
|  | $15 x^{2}-8 x+1$ | A1 |  |
|  | $(x+2)(3 x-1)(5 x-1)$ oe, cao | A1 | If zero scored, SC1 for an answer of $(x+2)(3 x-1)(5 x-1)$ without working. |
| 4(a) | ${ }^{5} C_{2} \times{ }^{8} C_{4}$ oe | M1 |  |
|  | 700 | A1 |  |
| 4(b) | $3 \times 6$ ! oe | M1 |  |
|  | 2160 | A1 |  |
| 5(a) | $4 \alpha-12=\alpha+3$ and $4-\beta=-2$ | M1 |  |
|  | $\alpha=5$ | A1 |  |
|  | $\beta=6$ | A1 |  |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 5(b) | $\sqrt{\text { their }(\alpha+3)^{2}+(-2)^{2}}$ | M1 |  |
|  | $\frac{2 \mathbf{j}-\text { their } 8 \mathbf{i}}{\sqrt{\text { their } 68}}$ | A1 | FT their $\alpha$ |
| 6 | $3 x^{2}+8 x+5=k x-7$ | M1 |  |
|  | $3 x^{2}+(8-k) x+12[=0]$ soi | A1 |  |
|  | $(8-k)^{2}-4(3)(12)$ | M1 |  |
|  | $k^{2}-16 k-80 * 0$ | M1 |  |
|  | Critical values: <br> -4 and 20 soi | A1 |  |
|  | $-4<k<20$ | A1 | Alternative method: <br> M1 for $k=6 x+8$ oe <br> M1 for $y=(6 x+8) x-7$ <br> M1 for $3 x^{2}+8 x+5=(6 x+8) x-7$ <br> A1 for $x= \pm 2$ <br> A1 for $k=-4, k=20$ <br> A1 for $-4<k<20$ |
| 7(a) | $\begin{aligned} & x+2 y=\lg 5 \text { or } \\ & 3 x+4 y=\lg 50 \end{aligned}$ | B1 |  |
|  | Solves their linear simultaneous equations | M1 |  |
|  | $x=\lg 2$ or equivalent simplified form | A1 |  |
|  | $y=\frac{1}{2} \lg \frac{5}{2}$ or equivalent simplified form | A1 | If A0 A0 then SC1 for a correct pair of unsimplified values or a correct pair of decimal values correct to at least 3 sf |
| 7(b) | $\left(x^{\frac{1}{3}}+2\right)\left(2 x^{\frac{1}{3}}-5\right)$ oe | M1 |  |
|  | $x^{\frac{1}{3}}=-2, \frac{5}{2}$ | M1 |  |
|  | $x=-8, \frac{125}{8}$ | A1 |  |
| 8(a) | $32-80 x+80 x^{2}-40 x^{3}+10 x^{4}-x^{5}$ | B3 | B2 for any four or five terms correct or <br> B1 for any three terms correct or M1 for a fully correct but unsimplified expansion |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 8(b) | Combines powers sufficiently to be able to take logs or applies correct log laws | M1 |  |
|  | For making use of their expansion from part (a) | M1 |  |
|  | $40 x^{2}(2-x)[=0]$ oe | M1 | FT their (a) if possible |
|  | $x=0, x=2 \mathrm{cao}$ | A1 |  |
| 9(a) |  | B3 | B1 for correct shape with three distinct linear sections <br> B1 for 3 and 1.6 on vertical axis B1 for $60,75,80$ on horizontal axis |
| 9(b) | $\begin{aligned} & 3 \times 60+15(1.6)+0.5(15)(1.4)+ \\ & 0.5(5)(1.6) \\ & \text { or } \\ & 3 \times 60+0.5(3+1.6)(15)+ \\ & 0.5(5)(1.6) \end{aligned}$ | M2 | M1 for attempting at least two terms of the sum: |
|  | 218.5 (metres) | A1 |  |
| 9(c) | $0.32\left(\mathrm{~ms}^{-2}\right)$ | B1 |  |
| 10(a) | $a=3, b=1$ | B2 | B1 for each |
| 10(b) | $\int_{0}^{\text {their } b} 4 x^{\frac{2}{3}} \mathrm{~d} x+\int_{\text {their } b}^{\text {their a }}(x-3)^{2} \mathrm{~d} x$ | M1 |  |
|  | $\left[\frac{3}{5} \times 4 x^{\frac{5}{3}}\right]_{0}^{\text {their } b}+\left[\frac{(x-3)^{3}}{3}\right]_{\text {their } b}^{\text {their a }}$ soi | M2 | M1 for each, soi |
|  | $\begin{aligned} & \frac{12}{5}(\text { their } b)-\frac{12}{5}(0)+ \\ & \frac{(\text { their } a-3)^{3}}{3}-\frac{(\text { their } b-3)^{3}}{3} \end{aligned}$ | M1 |  |
|  | $\frac{76}{15}$ or $5 \frac{1}{15}$ or <br> 5.07 or 5.06 rot to four or more figs; cao | A1 |  |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 11(a) |  | B3 | B1 for correct shape of $f$ or $f^{1}$ B1 for symmetry B1 for drawn over correct domain Maximum of 2 marks if not fully correct |
| 11(b)(i) | $[ \pm] \sqrt{x-1}=y-4$ soi | M1 |  |
|  | $g^{-1}(x)=4-\sqrt{x-1}$ | A1 |  |
|  | [Range] g ${ }^{-1} \leqslant 4$ | B1 |  |
|  | [Domain] $x \geqslant 1$ | B1 |  |
| 11(b)(ii) | $\ln \left(2\left[(x-4)^{2}+1\right]+1\right)$ | M1 |  |
|  | $\ln \left(2 x^{2}-16 x+35\right)$ | A1 |  |
| 11(b)(iii) | Valid explanation, e.g. some of the values in the range of f are outside the domain of $g$ | B1 |  |
| 12(a) | $\frac{\mathrm{d}\left(\mathrm{e}^{3 x}\right)}{\mathrm{d} x}=3 \mathrm{e}^{3 x}$ | B1 |  |
|  | $\frac{\mathrm{d}(2 x+3)^{6}}{\mathrm{~d} x}=k(2 x+3)^{5}$ | M1 |  |
|  | $\begin{aligned} & \text { their }\left(3 \mathrm{e}^{3 x}\right)(2 x+3)^{6}+ \\ & \left.\left(\mathrm{e}^{3 x}\right) \text { their } 12(2 x+3)^{5}\right) \end{aligned}$ | M1 |  |
|  | $\left(3 \mathrm{e}^{3 x}\right)(2 x+3)^{6}+\left(\mathrm{e}^{3 x}\right)\left(12(2 x+3)^{5}\right)$ | A1 |  |
|  | $\left(3 \mathrm{e}^{3 x}\right)(2 x+3)^{5}(2 x+7)=0$ | M1 |  |
|  | $x=-1.5,-3.5$ | A1 |  |
| 12(b) | $\begin{array}{ll} x=0.5 & \mathrm{f}^{\prime \prime}(0.5)[=-5]<0 \Rightarrow \max \\ x=3 & \mathrm{f}^{\prime \prime}(3)[=5]>0 \Rightarrow \min \end{array}$ | B2 | B1 for either one correct |


| Question | Answer | Marks | Partial Marks |
| :---: | :--- | ---: | :--- |
| $12(\mathrm{c})$ | $h=\frac{10}{x^{2}}$ | B1 |  |
|  | $S=8 x^{2}+10 x\left(\right.$ their $\left.\frac{10}{x^{2}}\right)$ | B1 |  |
|  | $\frac{\mathrm{d} S}{\mathrm{~d} x}=16 x-100 x^{-2}$ oe | M1 |  |
|  | $16 x-100 x^{-2}=0, x=\sqrt[3]{\frac{25}{4}}$ oe | A1 | FT their $\frac{\mathrm{d} S}{\mathrm{~d} x}=0$ if possible |
|  | 81.4 or $81.4325 \ldots$ rot to four or more <br> figs | A1 |  |

